Math 216 Exam 1 Solutions
Fall 2006 - Hartlaub.

#1 (a) The true value is 105 picocuries per liter of radon. Confidence intervals can be used to test 2 sided alternatives. Thus, to test $H_0: \theta = 105$ vs $H_1: \theta \neq 105$, where θ is the median amount of radon detected in a population of homes exposed to this amount of radon, we simply check if 105 is in the C.I. Since the true value, 105, is contained in all 4 of the CIs provided, we would not be able to reject H_0. That is, the CIs provide plausible values for θ and since 105 is in all of the intervals, the detectors appear to be working fine.

(b). The estimated median for the sign procedure (102.8) is simply the median of the radon readings in the 12 homes. The estimated median for the signed rank procedure (103.2) is the median of the Walsh averages. i.e. the median of all pairwise averages $\frac{x_i + x_j}{2}$ $1 \le i \le j \le 12$.

(c) I would use one of the CIs for the sign procedure because the distribution of the readings from the detectors in the 12 homes is not symmetric. The distribution is right-skewed with no outliers. We are 96.14% confident that the median amount of radon detected is between 96.6 and 111.4 picocuries per liter.

Math 216 Exam 1 Solutions
Fall 2006 - Hartlaub

#2. (a) We examine the heart-rate increase (Diff-low = lowfinal - lowrest) from low-rate exercise. Let θ = median of the differences for low-rate exercise. With only 5 observations it is very difficult to assess the shape of a distribution. (Either the sign test or the signed rank test will be accepted, as long as the student states the correct assumptions.

* Sign Test: $B = 4$
p-value = .0625

Signed Rank: $T^+ = 10$ (Assume symmetry)
p-value = .05

↳ This is near the borderline of significance: with only 4 nonzero differences it is fairly strong evidence that heart rate increases but (barely) not significant at the 0.05 level!

(b) We must first find the final - Resting differences for both exercise rates. Use diff-low from part (a) and let Diff_Med = medfinal - medrest. Then compute the differences of the differences; let diff = diff-med - diff-low. Apply the Wilcoxon signed rank test or the sign test to this final set of differences (diff.) (NOTE: The Wilcoxon rank sum test is not appropriate because we do not have independent samples.) The hypotheses are: $H_0: \theta_{diff} = 0$ vs $H_1: \theta_{diff} > 0$.

Test stat: MWW: $T^+ = 15$ p-value: 0.030

Sign Test: $B = 5$ p-value = 0.0313.

For either procedure, we reject H_0 (since .03 or .0313 < .05) and conclude medium-rate exercise increases are significantly greater

Math 216 Exam 1 Solutions

Fall 2006 Hartlaub

#3. (a) $H_0: \eta_{OJ} = \eta_{ACID}$ vs $H_1: \eta_{OJ} > \eta_{ACID}$

or: $OJ = ACID + \Delta$ $\quad H_0: \Delta = 0$ vs $H_1: \Delta > 0$.

Test stat: $W = 135$ $\boxed{p\text{-value} = .0129}$ MWW.

At $\alpha = .05$, we reject H_0 (since $.0129 < .05$) and conclude that OJ tends to produce larger ondontoblasts.

(b) NOTE: Minitab output taken from another text does not match the Minitab output if you compute the CI with Minitab yourself. You did not need to do this!

All $m \times n = 10 \times 10 = 100$ differences $\{Y_j - X_i\}$ were computed and sorted. Then the null distribution of the Wilcoxon rank sum statistic was used to find the position constant (how many positions to count up from the bottom + down from the top.)

$$1 - \alpha = .936 \Rightarrow \alpha = .064 \Rightarrow \alpha/2 = .032$$

$$L_{.064} = \frac{10((2)(10) + 10 + 1)}{2} + 1 - W_{.032} \leftarrow \text{Table A.6}$$

$$= 155 + 1 - 130 = 26$$

93.6% CI is $\left(U^{(26)}, U^{(100+1-26)}\right) = \left(U^{(26)}, U^{(75)}\right)$

$= (-.104, 9.098)$ according to incorrect output.

(c) The null hypothesis in part (a) was rejected because the one sided p-value is below the significance level of $\alpha = .06$. A 93.6% CI is equivalent to an $\alpha = 1 - .936 = .064$ level two sided test. The duality between inferences for one sided tests and the bound on intervals only holds for confidence bounds.

Math 216 Exam 1 Solutions

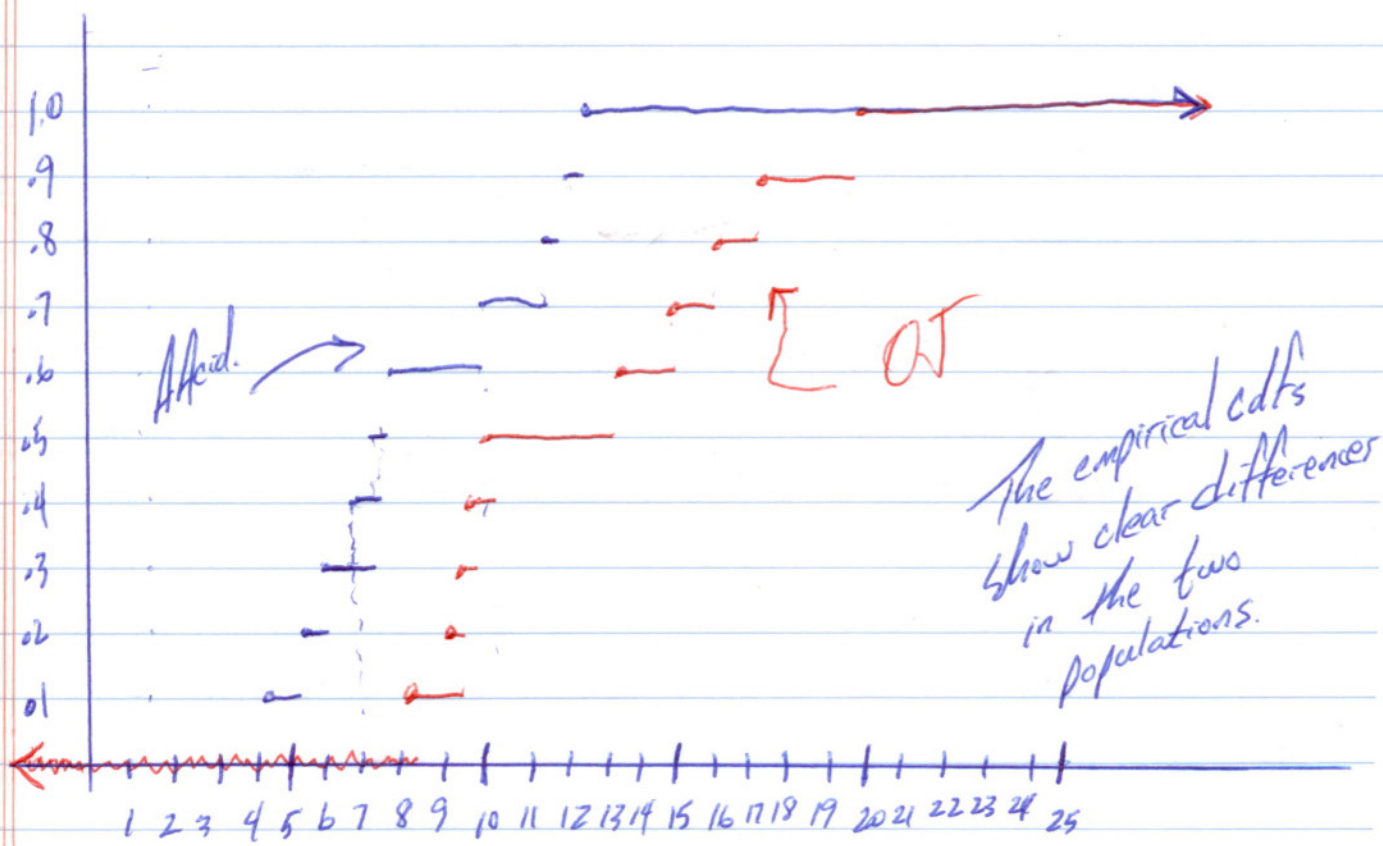
Fall 2006 Har-Haub

#3 (d) The empirical c.d.f.'s are:

OJ

$$F_m(t) = \begin{cases} 0, & t < 8.2 \\ 1/10, & 8.2 \le t < 9.4 \\ 2/10, & 9.4 \le t < 9.6 \\ 3/10, & 9.6 \le t < 9.7 \\ 4/10, & 9.7 \le t < 10 \\ 5/10, & 10 \le t < 14.5 \\ 6/10, & 14.5 \le t < 15.2 \\ 7/10, & 15.2 \le t < 16.1 \\ 8/10, & 16.1 \le t < 17.6 \\ 9/10, & 17.6 \le t < 21.5 \\ 1, & 21.5 \le t \end{cases}$$

$$G_n(t) = \begin{cases} 0, & t < 4.2 \\ 1/10, & 4.2 \le t < 5.2 \\ 2/10, & 5.2 \le t < 5.8 \\ 3/10, & 5.8 \le t < 6.4 \\ 4/10, & 6.4 \le t < 7 \\ 5/10, & 7 \le t \le 7.3 \\ 6/10, & 7.3 \le t < 10.1 \\ 7/10, & 10.1 \le t < 11.2 \\ 8/10, & 11.2 \le t < 11.3 \\ 9/10, & 11.3 \le t < 11.5 \\ 1, & 11.5 \le t \end{cases}$$



The empirical cdfs show clear differences in the two populations.

Math 216 Exam 1 Solutions
Fall 2006 Hartlaub

#4 (a) Fligner - Policello

(b) Mann - Whitney - Wilcoxon or Wilcoxon rank sum

(c) Since the ratio of the 2 standard deviations is $\frac{11.83}{8.43} = 1.4 < 2$, I would use the MWW procedure. (Students could also argue that Kolmogorov - Smirnov should be used to detect <u>any</u> differences in the two populations.)

#5. Similarities:

- Both test $H_0: \gamma^2 = 1$ vs. $H_1: \gamma^2 \neq 1$ or $\gamma^2 > 1$ or $\gamma^2 < 1$
- Both assume
  1. X's are a R.S. from continuous Pop 1
     Y's are a R.S. from continuous Pop 2
  2. X's and Y's are mutually independent.
- Both use normal critical values for large samples.

Differences:

Ansari - Bradley	Jackknife
- Assume equal medians (unrealistic)	- Assume finite 4^{th} moment
- Distribution free	- Asymptotically distribution free
- Based on scoring function ("easy")	- Computationally intensive
- Ties correction is needed	- Ties correction is not needed.